What Happened to the Cost? Teacher Notes

Topic Area: Matrices

NCTM Standard:

- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices.

Objective

Given several data tables, the student will be able to create a matrix to represent the data, perform operations using the matrices, and apply the results to problem solving tasks.

Getting Started

Have the students work in pairs or in small groups to determine what information would be needed to calculate the cost of manufacturing an item for retail sales and what would influence these expenses.

Prior to using this activity:

- Students should have a basic understanding of the properties of matrices.
- Students should be able to determine if two matrices can be multiplied and perform the operation.
- Students should be able to understand the meaning of the resulting matrix.
- Students should be able to understand and calculate percent in relationship to profit and loss.

Ways students can provide evidence of learning:

- The student will be able to create an appropriate matrix to represent the data, given a table of data.
- The student will be able to multiply the matrices and analyze the results, given two matrices.

Common calculator or content errors students might make:

- Students may create a matrix that does not accurately represent the data.
- Students may multiply matrices in the wrong order, resulting in a single number rather than a list of numbers.

Definitions:

- Matrix
- Cell
- Percent

What Happened to the Cost? "How To"

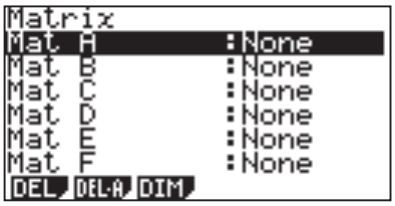
The following will demonstrate how to create a matrix and enter the values of the cells into the Casio *fx-9750GII*, to recall a matrix, and to perform operations with matrices.

Orchard	Apples	Pears	Peaches
Farm 1	125	110	135
Farm 2	205	95	185
Farm 3	158	82	170

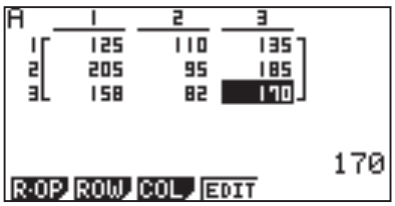
Price/Box
$29
$22
$27

To create a matrix for the table above:

1. From the Main Menu, highlight the RUN•MAT icon and press [EXE] or press [1].
   Press [F1] to access the matrix editor.

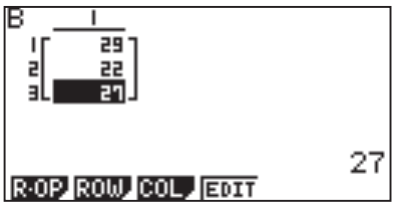


2. To clear any data from previous matrices, press [F2] (DEL-A), then [F1] (Yes).



3. To create a 3 x 3 matrix, press:
   [▶] [3] [EXE] [3] [EXE] [EXE].
   Enter the values for each cell and press [EXE].
   The screen should look like the one to the right.

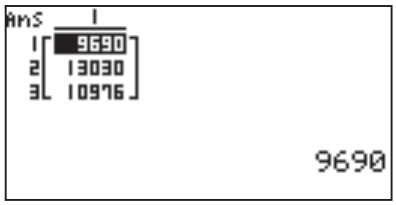
4. To enter the second matrix, press:
   [EXIT] [▼] [▶] [3] [EXE] [EXE] for a 3 x 1 matrix.
   Enter the values using the method described in step 3. The screen should like the one to the right.



To multiply the two matrices:

1. Press [EXIT] twice to return to the RUN-MAT screen.

2. Press [OPTN] [F2] [F1] [ALPHA] [X,θ,T] [×] [F1] [ALPHA] [log] [EXE] to multiply the two matrices. The results are shown on the screen to the right.



What Happened to the Cost? Activity

Ever wonder why it cost so much to buy your favorite sport jersey? The amount of material it would take to make the shirt does not seem to match the price. Some talented people can make a great jersey and save themselves money while others are willing to spend the money and save the time for other things. In this activity, you will explore the costs involved in manufacturing an athletic jersey and discover why it costs so much more to buy one than make it yourself.

In this activity, you will use the given data tables and matrices to calculate the cost of materials, production, labor, and advertising and apply these costs to the final price of a football, basketball, and baseball jersey. The company is manufacturing 2000 football jerseys, 1500 basketball jerseys, and 1200 baseball jerseys. The cost for the materials needed for each jersey is $5.99, $3.99, and $4.99, respectively. Using your calculations, you will then determine the selling price for each shirt in order to make a profit.

Production Costs			
Sport	**Cutting**	**Assembly**	**Packaging**
Football	$0.25/hr.	$1.75/hr.	$0.08/hr.
Basketball	0.10/hr.	0.50/hr.	0.05/hr.
Baseball	0.15/hr.	0.75/hr.	0.05/hr.

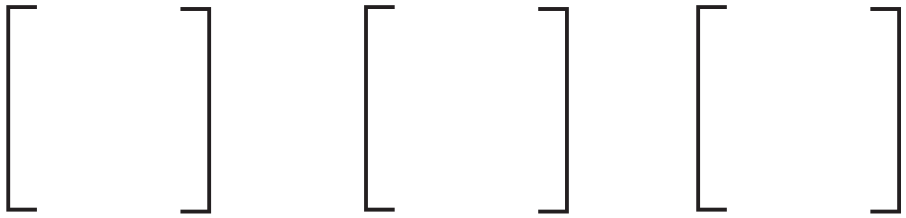
Labor Costs			
Sport	**Cutting**	**Assembly**	**Packaging**
Football	$1.88/hr.	$10.31/hr.	$0.58/hr.
Basketball	0.75/hr.	4.13/hr.	0.36/hr.
Baseball	1.13/hr.	6.19/hr.	0.36/hr.

Advertising Costs			
Sport	**Photography**	**Copy**	**Model**
Football	$2.25/hr.	$0.38/hr.	$22.50/hr.
Basketball	3.00/hr.	0.50/hr.	24.00/hr.
Baseball	3.75/hr.	0.63/hr.	20.83/hr.

Questions

1. Create a matrix for the production costs, the labor costs, the advertising costs, and the number of each type of jersey to be manufactured. Multiply each of the matrices for the various costs by the number of jerseys. Fill in the resulting matrices. What is the meaning for each of the values in the matrices?

Production Costs Labor Costs Advertising



_____

_____

2. Calculate the total cost for material for each jersey. Explain why this cannot be done using matrices.

_____

_____

3. Calculate the total cost for manufacturing a football jersey. What are some reasons why production and labor costs are higher for this jersey than for the others?

_____

_____

4. Find the cost of manufacturing each baseball jersey; is it more or less than the football jersey? Does this seem like a reasonable cost? What would account for the difference?

_____

_____

5. Calculate the total cost for manufacturing a basketball jersey. What are some of the reasons why advertising costs are higher for this jersey than for the others?

_____

_____

6. If each jersey sells for $55.00, create a matrix that can be used to find the product of the cost per jersey and the number of jerseys to be sold. Record your answer in the space provided. Multiply this matrix by the matrix for the total number being manufactured. What is the projected total from the sales of the jerseys? What will be the percentage of profit from the sales?
[ ]

_____

_____

Extensions

1. Explain how the sizes of two matrices determine whether they can be multiplied. In order to multiply a 2 x 3 matrix and a 4 x 2 matrix, which one should be first? Give the dimensions of the resulting matrix.

_____

_____

2. The school store is selling shirts with short sleeves, shirts with long sleeves, and shirts with hoods. Short sleeve shirts sell for $15.00 each, long sleeve shirts sell for $17.50 each, and hooded shirts sell for $20.00 each. Below are the matrices that were set up to calculate the total sales.

Number of Shirts
[200 155 125 **]**

Cost per Shirt
[15.00 17.50 20.00 **]**

a) Explain why an error message was given when these matrices were multiplied using a calculator.
b) Explain how to work the problem in the calculator, in order to obtain a correct answer for total sales overall.

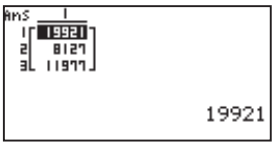
_____

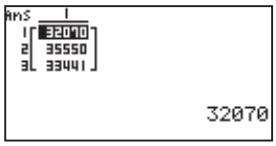
_____

_____

Solutions

1. **Production Cost** **Labor Cost** **Advertising Cost**





For each matrix, row 1 is the total cost for football jerseys, row 2 is the total cost for basketball jerseys, and row 3 is the total cost for baseball jerseys.

2. **Football:** 2000 ($5.99) = **$11,980.00**
Basketball: 1500 ($3.99) = **$5,985.00**
Baseball: 1200 ($4.99) = **$5,988.00**

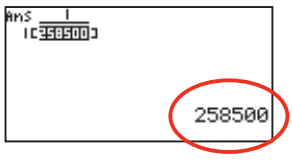
Multiplying a 3 x 1 matrix by a 1 x 3 matrix would result in a 3 x 3 matrix which would not give a correct response. Multiplying a 1 x 3 matrix by a 3 x 1 matrix would result in giving the total cost of materials for all of the jerseys.

3. -The total cost is $3,221 + $19,921 + $32,070 + $11,980 which equals **$67,192**.
-Answers may vary. Some reasons would include the cost of materials are higher, the amount of time for construction is higher, and the cost of advertising due to endorsements by athletes who are paid higher salaries.

4. -The total cost is $1,485 + 11,977 + 33,441 + 5,988 which equals **$52,891**.
-This cost is less than a football jersey.
-Answers will vary according to experience.
-Some of the difference is the number of jerseys being made and the extra materials.

5. -The total cost is $1,485 + 8,127 + 35,550 + 5,985 which equals **$51,147**.
-One answer may be that although endorsements may not cost as much due to popularity, the cost per jersey is more since there are less being manufactured.

6. **[** 55 55 55 **]**



Percentage of Profit:
[258,500 – (67,192 + 52,891 + 51,147)] /258,500 = .3376 or **33.8%**

Extension Solutions

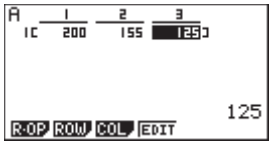
1. The dimensions of the matrices must be such that the number of columns in the first matrix is the same as the number of rows in the second matrix.

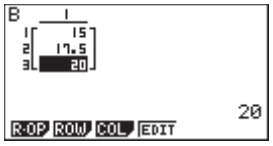
   The 4 x 2 matrix would be entered first followed by the 2 x 3 matrix. The resulting dimensions would be a 4 x 3 matrix.

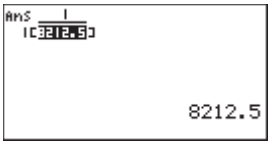
2. a) **Matrix A** = **[** 200 155 125 **]** **Matrix B** = **[** 15.00 17.50 20.00 **]**

   Matrix AB = **[** 200(15) + 155(?) 200(17.50) 200(20.00) **]**

   b) Since the first cell of AB must contain the product of $cell_{1,1}$ + $cell_{2,2}$ and there is not a second row, these matrices will not work. The second matrix would need to be rewritten as a 3 x 1 matrix in order to calculate the total.







Squaring Up Sides — Teacher Notes

Topic Area: Properties of Squares

NCTM Standards:

- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.

Objective

The student will be able to use algebra and statistics to prove that a parallelogram is a rectangle or square, demonstrate that the diagonals of a rectangle are equal, demonstrate that all four sides of a square are equal, demonstrate that the diagonals of a square are perpendicular to each other, and apply the properties of rectangles and squares to real-life problems.

Getting Started

As a class, review the meaning of slope and the slope-intercept form of an equation; include in the discussion the properties of a parallelogram. Review methods of proving triangles congruent using the Hypotenuse-Leg method.

Prior to using this activity:

- Students should be able to find the xy-line for a pair of coordinates using a graphing calculator.
- Students should be able to perform calculations involving square roots and trigonometric functions using a graphing calculator.

Ways students can provide evidence of learning:

- The student will be able to write conjectures pertaining to squares.
- The student will be able apply the properties of a parallelogram to real-life problems.

Common mistakes to be on the lookout for:

- Students may confuse the x and y values in the calculations.
- Students may enter the problem incorrectly into the calculator.

Definitions

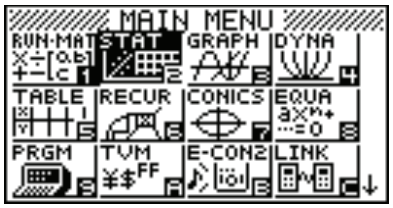
- Parallelogram
- Rectangle
- Square
- Perpendicular
- Endpoint
- Slope
- Diagonal
- Midpoint
- Congruent
- Hypotenuse
- Leg

Squaring Up Sides "How-To"

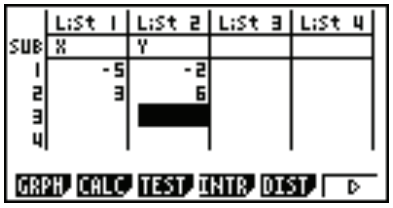
The following will demonstrate how to enter a set of coordinates into two lists using the Statistics mode of the Casio *fx-9750GII*. After the list is set up, you will find the slope of a line containing the points, save the equation in the Graph mode, and find the intersection of two lines.

Line segment AB has endpoints at (–5, –2) and (3, 6) and segment CD has endpoints at (–6, 4) and (3, –7). Find the slope for each line segment and the coordinates of their intersection.

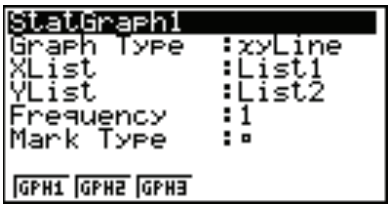
To enter values into a list and find the line of best fit:



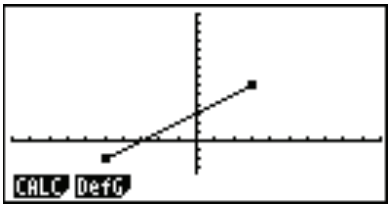
1. From the Main Menu, highlight the Statistics icon and press EXE or press 2.
2. To label the first column, highlight the space below **List 1** and press ALPHA + (X) EXE.
3. To label the second column, highlight the space below **List 2** and press ALPHA − (Y) EXE.



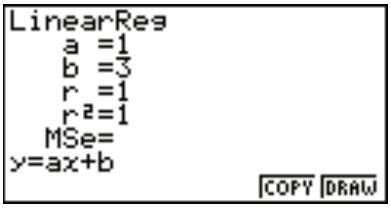
4. Enter the x-values into **List 1** and the y-values into **List 2**. Be sure to press EXE after each value.
5. To view the points, press F1 (GRPH) F6 (Set) ▼ F1 (GPH1) ▼ F2 (XY) ▼ F1 (List) 1 ▼ F1 (List) 2 EXE F1 (GPH1).



6. Press EXIT and F1 (GPH1) to view the graph.

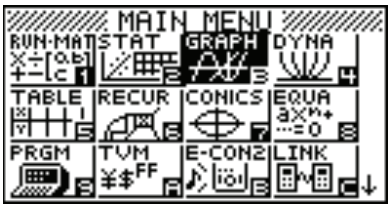


7. Press F1 (Calc) F2 (X) F1 (ax+b) to find the line of best fit.

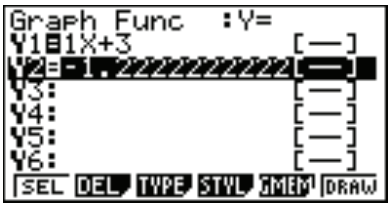


8. Press F5 (Copy) EXE to copy the equation into the graph function.
9. Repeat the same steps to find the equation for the second segment.

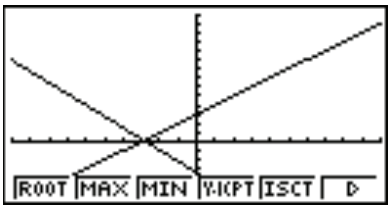
To graph the two equations and find the intersection:



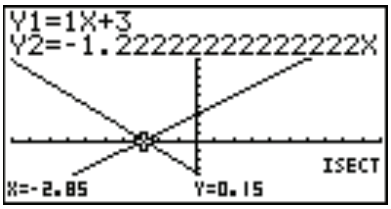
1. From the Main Menu, highlight the Graph icon and press EXE or press 3.



2. To graph the two equations, highlight each equation and press F1 (Sel) to turn the function on; when the equal signs are highlighted, you know the equation is selected. Then press F6 (Draw).



3. While viewing the graph, press F5 **(G-Solv)** F5 (ISCT) to find the intersection of the two equations.



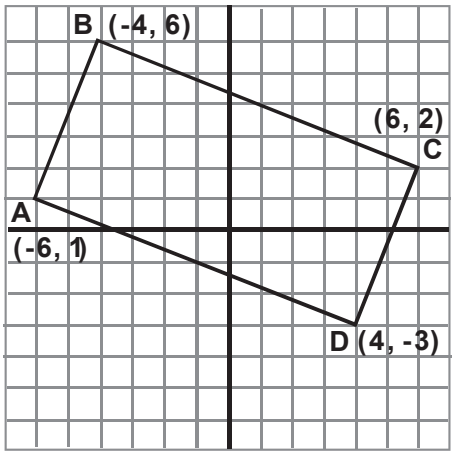
4. The coordinates are displayed at the bottom of the screen.

Squaring Up Sides — Activity

Two special quadrilaterals that are also parallelograms are the rectangle and the square. These are figures that are seen everywhere. In the activity, we will determine what makes them so special and solve problems that involve those special properties.

Questions

The diagram at the right shows ▱ABCD. By definition, a rectangle is a quadrilateral with four right angles.



1. Find the equation of the following line segments
   a. $\overline{AD}$

   _____

   b. $\overline{AB}$

   _____

2. Are these segments perpendicular?

   _____

3. Are all four angles equal to 90°? Explain.

   _____

   _____

4. Find the lengths of the following segments to the nearest tenth.
   a. $\overline{AC}$

   _____

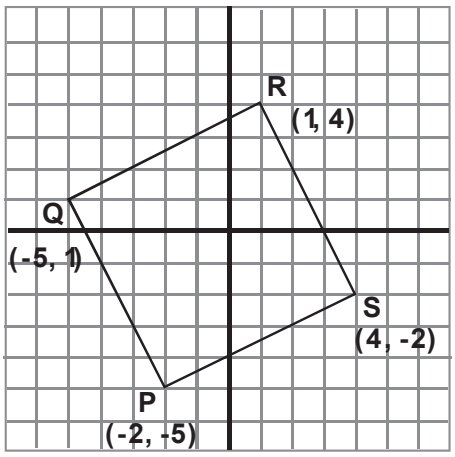
   b. $\overline{BD}$

   _____

5. What does this tell you about the diagonals of a rectangle?

   _____

The diagram below shows ▱PQRS. By definition, a square is a quadrilateral with four congruent sides and four congruent angles.



6. Find the lengths of the following segments to the nearest tenth.

   a. $\overline{PQ}$

   _____

   b. $\overline{QR}$

   _____

7. If a square is a special parallelogram, then are all four sides congruent? Explain.

   _____

   _____

8. Find the equation of the following segments.

   a. $\overline{PQ}$

   _____

   b. $\overline{QR}$

   _____

9. Are these segments perpendicular? If so, does that make this figure a square?

   _____

10. Find the equation of the following segments.

    a. $\overline{QS}$

    _____

    b. $\overline{PR}$

    _____

11. What can you conclude about the diagonals of a square?

    _____

A gazebo is being placed in the middle of a town park that is rectangular in shape. The plan was drawn on a grid showing the corners of the park at (–4, 1), (2, 5), (4, 2), and (–2, –2).

12. Is the park truly rectangular in shape? Explain.

_____

_____

13. Where will the center of the gazebo be located on the grid?

_____

14. What is the perimeter of the park to the nearest tenth?

_____

The Better Built construction company is building a deck for a customer. The deck is to measure 28 ft. by 16 ft.

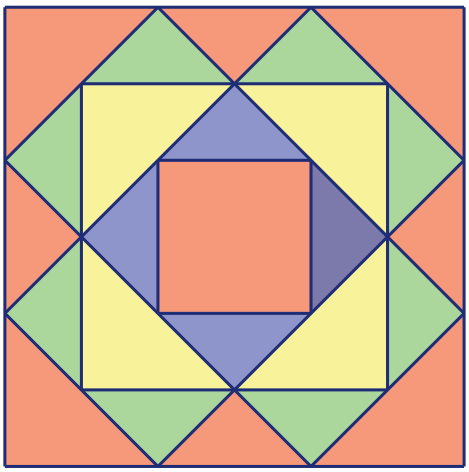
15. How would they ensure that the deck is square (has all right angles)?

_____

16. What would this measure be to the nearest hundredth?

_____

Mrs. Santiago is creating a quilt using the block pattern, shown below. Each block will be 8 in. on a side. In this pattern, the red square measures 4 in. on a side.



17. If the red square touches the blue square at its midpoints, what is the length of each side of the blue square?

_____

18. The blue square touches the yellow square at its midpoints. What is the length of the side of the yellow square?

_____

19. The finished quilt will measure 6 blocks by 8 blocks. What will be the perimeter of the finished quilt before the binding is sewn around the quilt?

_____

A local youth center is planning to build a baseball field in a nearby field. The distance between home plate and first base will be 60 ft.

20. What is the distance between home plate and second base to the nearest tenth?

_____

21. If a player makes a home run, how far will they run?

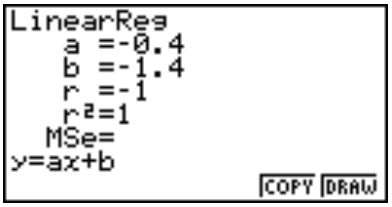
_____

22. If the pitcher is 3 ft. further away from home plate than the center of the infield, how far away would the pitcher be from the batter to the nearest tenth?

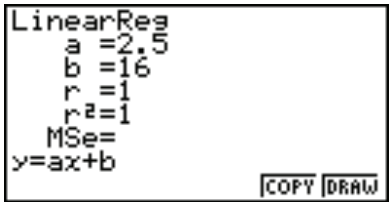
_____

Solutions

1. a. y = –0.4x – 1.4



b. y = 2.5x + 16

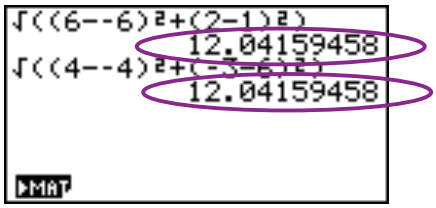


2. Yes; 2.5 is the negative reciprocal of –0.4.

3. Yes. Since the figure is a parallelogram, the opposite sides are parallel making the consecutive angles supplementary and the opposite angles are equal.

4. a. $\overline{AC}$ = 12.04

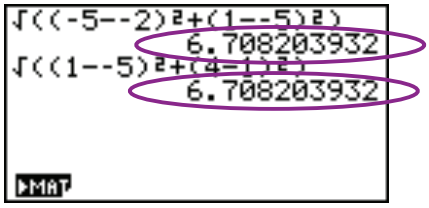
b. $\overline{BD}$ = 12.04



5. The diagonals are equal.

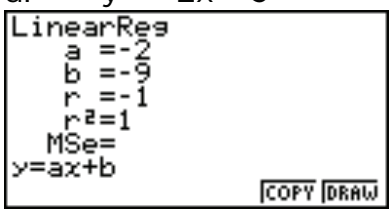
6. a. $\overline{PQ}$ = 6.7

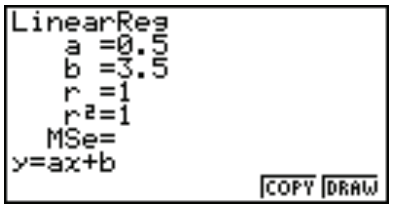
b. $\overline{QR}$ = 6.7



7. Yes; since opposite sides are equal, all the sides would equal 6.7.

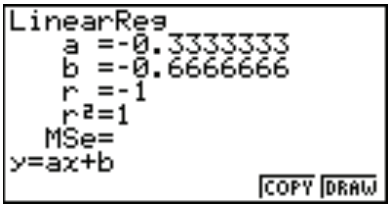
8. a. y = –2x – 9



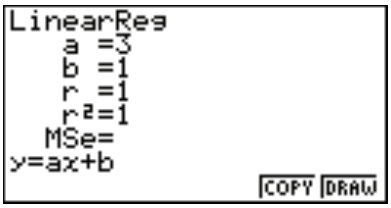


9. Yes; all four sides are congruent and all four angles equal 90°.

10. a. y = –0.33x – 0.66

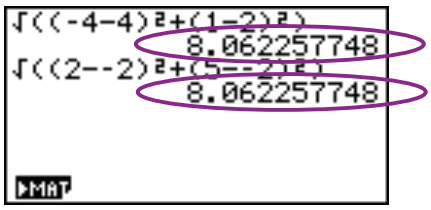


b. y = 3x + 1

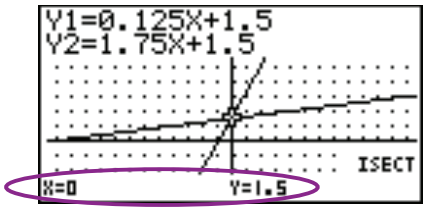


11. The diagonals are perpendicular.

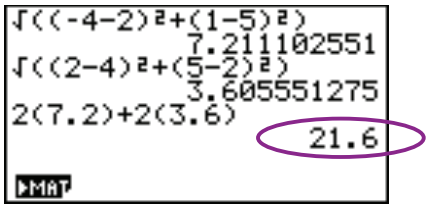
12. Yes; the diagonals of the quadrilateral are equal.



13. (0, 1.5)

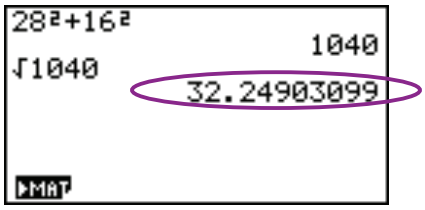


14. 21.6 units

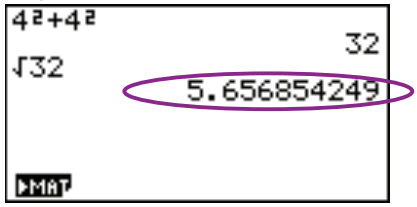


15. Measure the diagonals to see that they are equal.

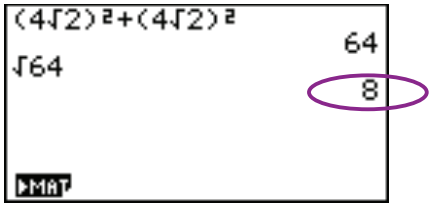
16. 32.2 feet



17. $4\sqrt{2}$

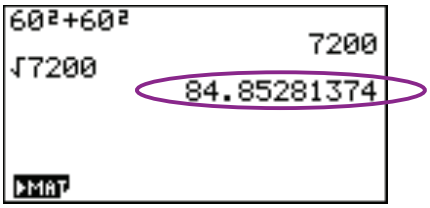


18. 8 inches



19. Length = 6(8) 48 in. Width = 8(8) = 64 in.
Perimeter = 2(48) + 2(64) = 224 in.

20. 84.9 ft.



21. 4(60) = 240 ft

22. 45.5 ft.